

# Mocninné řady

Nalezněte poloměr konvergence mocninné řady a vyšetřete body  $x = \pm r$ :

1.  $\sum_{n=1}^{\infty} \frac{x^n}{n^p}$
2.  $\sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{n} (x+1)^n$
3.  $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$
4.  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2} x^n$
5.  $\sum_{n=1}^{\infty} \left[ \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \right]^p \left(\frac{x-1}{2}\right)^n$
6.  $\sum_{n=1}^{\infty} \frac{m \cdot (m-1) \cdots (m-n+1)}{n!} x^n$ ,  $m = \text{konstanta}$
7.  $\sum_{n=1}^{\infty} \frac{x^n}{a^n + b^n}$ ,  $a, b > 0$
8.  $\sum_{n=1}^{\infty} \frac{x^n}{a^{\sqrt{n}}}$ ,  $a > 1$
9.  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right) x^n$
10.  $\sum_{n=1}^{\infty} \frac{(3 + (-1)^n)^n}{n} x^n$
11.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \left(\frac{n}{e}\right)^n x^n$
12.  $\sum_{n=1}^{\infty} \frac{x^{n^2}}{2^n}$

Nalezněte rozvoj funkce v mocninnou řadu, určete interval, kde platí rovnost:  
(není-li uvedeno jinak, je  $b = 0$ )

1.  $f(x) = e^{-x^2}$

2.  $f(x) = \cos^2 x$

3.  $f(x) = \sin^3 x$

4.  $f(x) = \frac{x^{10}}{1-x}$

5.  $f(x) = \frac{1}{(1-x)^2}$

6.  $f(x) = \ln \sqrt{\frac{1+x}{1-x}}$

7.  $f(x) = \frac{x}{(1-x)(1-x^2)}$

8.  $f(x) = \operatorname{arctg} x$

9.  $f(x) = \arcsin x$

10.  $f(x) = \operatorname{argsinh} x$

(tj.  $\ln(x + \sqrt{1+x^2}) =$  )

11.  $f(x) = (1+x) \cdot \ln(1+x)$

12.  $f(x) = \frac{1}{4} \ln \frac{1+x}{1-x} + \frac{1}{2} \operatorname{arctg} x$

13.  $f(x) = \operatorname{arctg} \frac{2-2x}{1+4x}$

14.  $f(x) = x \cdot \operatorname{arctg} x - \ln \sqrt{1+x^2}$

15.  $f(x) = x \cdot \arcsin x + \sqrt{1-x^2}$

16.  $f(x) = x \cdot \ln(x + \sqrt{1+x^2}) - \sqrt{1+x^2}$

17.  $f(x) = \operatorname{argtgh} x$

(tj.  $\frac{1}{2} \ln \frac{1+x}{1-x} =$  )

18.  $f(x) = \arcsin 2x$

19.  $f(x) = \operatorname{arctg} x^2$

20.  $f(x) = \frac{1}{12} \ln \frac{1+3x}{1-3x} + \frac{1}{6} \operatorname{arctg} 3x$

21.  $f(x) = \operatorname{arctg} 3x$
22.  $f(x) = \operatorname{arctg} \frac{6x}{1-9x^2}$
23.  $f(x) = x \cdot \operatorname{arctg} 2x - \frac{1}{2} \ln \sqrt{1+4x^2}$
24.  $f(x) = \ln(1+x-2x^2)$
25.  $f(x) = x \cdot \arcsin 3x + \frac{1}{3} \sqrt{1-9x^2}$
26.  $f(x) = x \cdot \ln(3x + \sqrt{1+9x^2}) - \frac{1}{3} \sqrt{1+9x^2}$
27.  $f(x) = x \operatorname{argsinh} 5x - \frac{1}{5} \sqrt{25x^2+1}$
28.  $f(x) = \frac{1}{8}x^2 - \frac{1}{32} \ln(4+16x^2)$
29.  $f(x) = \ln(2x + \sqrt{1+4x^2})$
30.  $f(x) = \operatorname{arctg} [4(x-2)^2]$  (střed v  $b = 2$ )
31.  $f(x) = \ln\left(\frac{1}{4} + x^2\right)$
32.  $f(x) = \arcsin 3x^2$
33.  $f(x) = \operatorname{argsinh} 2(x+1)$  (střed v  $b = -1$ )
34.  $f(x) = \frac{1}{8}x^2 - \frac{1}{16} \operatorname{arctg} 2x^2$
35.  $f(x) = x \arcsin 4x + \frac{1}{4} \sqrt{1-16x^2}$
36.  $f(x) = \frac{1}{4} \operatorname{arctg} 2x + \frac{1}{2} \frac{x}{1+4x^2}$

# Mocninné řady

## Výsledky

Nalezněte poloměr konvergence mocninné řady a vyšetřete body  $x = \pm r$ :

1.  $r = 1; (-1, 1)$   
 $x = -1$  : KA, je-li  $p > 1$ ; K, je-li  $0 < p \leq 1$ .  
 $x = 1$  : KA, je-li  $p > 1$ ; D, je-li  $p \leq 1$
2.  $r = \frac{1}{3}; (-\frac{4}{3}, -\frac{2}{3})$   
 $x = -\frac{4}{3}$  : K ;  $x = -\frac{2}{3}$  : D
3.  $r = 4; (-4, 4)$   
 $x = \pm 4$  : D
4.  $r = \frac{1}{e}; (-\frac{1}{e}, \frac{1}{e})$   
 $x = \pm \frac{1}{e}$  : D
5.  $r = 2; (-1, 3)$   
 $x = -1$  : KA, je-li  $p > 2$ ; K, je-li  $0 < p \leq 2$   
 $x = 3$  : KA, je-li  $p > 2$ ; D, je-li  $p \leq 2$
6.  $r = 1; (-1, 1)$   
 $x = -1$  : KA, je-li  $m \geq 0$ ; D, je-li  $m < 0$   
 $x = 1$  : KA, je-li  $m \geq 0$ ; K, je-li  $-1 < m < 0$
7.  $r = \max\{a, b\}; (-r, r)$   
 $x = \pm r$  : D
8.  $r = 1; (-1, 1)$   
 $x = \pm 1$  : KA
9.  $r = 1; (-1, 1)$   
 $x = \pm 1$  : D
10.  $r = \frac{1}{4}; (-\frac{1}{4}, \frac{1}{4})$   
 $x = \pm \frac{1}{4}$  : D
11.  $r = 1; (-1, 1)$   
 $x = -1$  : D ;  $x = 1$  : K

12.  $r = 1; (-1, 1)$   
 $x = \pm 1 : \text{KA}$

Nalezněte rozvoj funkce v mocninnou řadu, určete interval, kde platí rovnost:  
(není-li uvedeno jinak, je  $b = 0$ )

1.  $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n} \quad (|x| < +\infty)$

2.  $1 + \sum_{n=1}^{\infty} (-1)^n \frac{2^{2n-1}}{(2n)!} x^{2n} \quad (|x| < +\infty)$

3.  $\frac{3}{4} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^{2n-1}}{(2n+1)!} x^{2n+1} \quad (|x| < +\infty)$

4.  $\sum_{n=10}^{\infty} x^n \quad (|x| < 1)$

5.  $\sum_{n=0}^{\infty} (n+1)x^n \quad (|x| < 1)$

6.  $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} \quad (|x| < 1)$

7.  $\frac{1}{2} \sum_{n=1}^{\infty} \left[ n + \frac{1-(-1)^n}{2} \right] x^n \quad (|x| < 1)$

8.  $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad (|x| \leq 1)$

9.  $x + \sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+1}}{2n+1} \quad (|x| \leq 1)$

10.  $x + \sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+1}}{2n+1} \quad (|x| \leq 1)$

11.  $x + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n(n+1)} \quad (|x| \leq 1)$

12.  $\sum_{n=0}^{\infty} \frac{x^{4n+1}}{4n+1} \quad (|x| < 1)$

13.  $\operatorname{arctg} 2 + \sum_{n=1}^{\infty} (-1)^n \frac{2^{2n-1}}{2n-1} x^{2n-1} \quad \left(-\frac{1}{4} < x \leq \frac{1}{2}\right)$   
 $\operatorname{arctg} 2 - \pi + \sum_{n=1}^{\infty} (-1)^n \frac{2^{2n-1}}{2n-1} x^{2n-1} \quad \left(-\frac{1}{2} \leq x < -\frac{1}{4}\right)$
14.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{2n(2n-1)} \quad (|x| \leq 1)$
15.  $1 + \frac{x^2}{2} + \sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n+2)!!} \frac{x^{2n+2}}{2n+1} \quad (|x| \leq 1)$
16.  $-1 + \frac{x^2}{2} + \sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!!}{(2n+2)!!} \frac{x^{2n+2}}{2n+1} \quad (|x| \leq 1)$
17.  $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} \quad (|x| < 1)$
18.  $2 \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!} 4^n \frac{x^{2n+1}}{2n+1} \quad \left(|x| \leq \frac{1}{2}\right)$
19.  $2 \sum_{n=0}^{\infty} \frac{(-1)^n}{4n+2} x^{4n+2} \quad (|x| \leq 1)$
20.  $\sum_{n=0}^{\infty} (-1)^n \frac{81^n}{4n+1} x^{4n+1} \quad \left(|x| < \frac{1}{3}\right)$
21. ???
22. ???
23. ???
24. ???
25. ???
26. ???
27. ???
28. ???
29. ???
30. ???

31. ???

32. ???

33. ???

34. ???

35. ???

36. ???