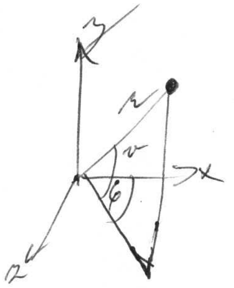
OBJEM KOOLE $\quad V = \frac{4}{3}\pi r^3$

$$M = \{(x, y, z) \in \mathbb{R}^3 ; x^2 + y^2 + z^2 \leq r^2\}$$

$$V = \int_M dx\, dy\, dz$$



$$x = r \cdot \cos\varphi \cdot \cos v$$
$$y = r \cdot \sin v$$
$$\cancel{x}\ z = r \cdot \sin\varphi \cdot \cos v$$

$$J = \begin{pmatrix} \cos\varphi \cdot \cos v & -r \cdot \cos\varphi \sin v & -r \overset{\sin}{\cancel{\cos}}\varphi \cos v \\ \sin v & r \cdot \cos v & 0 \\ \cos v \sin\varphi & -r \sin\varphi \sin v & r \cdot \cos\varphi \cdot \cos v \end{pmatrix}$$

$$\det(J) = r^2 \cos^2\varphi \cos^3 v + r^2 \sin^2\varphi \cdot \cos v \cdot \sin^2 v +$$
$$+ r^2 \cdot \sin^2\varphi \cdot \cos^3 v + r^2 \cdot \cos^2\varphi \sin^2 v \cdot \cos v =$$
$$= r^2 \cdot (\cos^3 v + \sin^2 v) = r^2 \cos v \neq 0$$

$$V = 8 \cdot \int_0^r \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} r^2 \cos v \; dr\, d\varphi\, dv =$$

$$= 8 \int_0^r r^2 \int_0^{\frac{\pi}{2}} \underbrace{\left[\sin v\right]_0^{\frac{\pi}{2}}}_{\sin\frac{\pi}{2} - \sin 0 = 1 - 0} dr\, d\varphi = 4\pi \int_0^r r^2 dr =$$

$$= 4\pi \left[\frac{r^3}{3}\right]_0^r = \frac{4\pi r^3}{3}$$

OBSAH KRUHU

πr^2

$~~K = \{(x,y) \in \mathbb{R}^2 ; (x-x_0)^2 + (y-y_0)^2 \leq r^2\}~~$

$K = \{(x,y) \in \mathbb{R}^2 ; x \in \langle x_0 - r ; x_0 + r \rangle , g_1(x) \leq y \leq g_2(x)\}$

$g_1(x) = y_0 - \sqrt{r^2 - (x-x_0)^2}$

$g_2(x) = y_0 + \sqrt{r^2 - (x-x_0)^2}$

$$\int_K 1 = \int_{x_0-r}^{x_0+r} \left(\int_{y_0-\sqrt{r^2-(x-x_0)^2}}^{y_0+\sqrt{r^2-(x-x_0)^2}} 1 \, dy \right) dx =$$

$$= \int_{x_0-r}^{x_0+r} 2 \sqrt{r^2-(x-x_0)^2} \, dx =$$

SUB: $x - x_0 = r \cdot \sin t$

$dx = r \cdot \cos t \, dt$

$$= 2 \int_{x_0-r}^{x_0+r} \sqrt{r^2-(x-x_0)^2} \, dx =$$

$$= 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \; r \cos t \, dt =$$

$$= 2r^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t \, dt = \underline{\underline{\pi r^2}}$$

POVRCH KOULE ($4\pi r^2$)

plocha v R^3 : $x^2+y^2+z^2=r^2$; $|P|=\iint_P dS$

$Q\{(x,y,z)\in P; z>0\}$

$|P|=2|Q|$

$Q=\{(x,y,z)\in R^3; (x,y)\in D; z=g(x,y)\}$

$D=\{(x,y)\in R^2; x^2+y^2<r^2\}$; $g(x,y)=\sqrt{r^2-x^2-y^2}$

$$|Q|=\iint_Q dS=\iint_D \sqrt{1+\left(\frac{\partial g}{\partial x}\right)^2+\left(\frac{\partial g}{\partial y}\right)^2}\,dx\,dy=$$

$$=\iint_D \sqrt{1+\frac{x^2}{r^2-x^2-y^2}+\frac{y^2}{r^2-x^2-y^2}}\,dx\,dy=\iint_D \frac{r}{\sqrt{r^2-x^2-y^2}}\,dx\,dy=$$

POLÁRNÍ SOUŘ:

$$=\int_{-\pi}^{\pi}\int_0^r \frac{r}{\sqrt{r^2-\varrho^2}}\,\varrho\,d\varrho\,d\varphi=2\pi r\left[-\sqrt{r^2-\varrho^2}\right]_0^r=\underline{\underline{2\pi r^2}}$$

$\int y^2 \, ds \quad ; \quad \varphi \in \langle 0; 2\pi)$

$\varphi_1(t) = a(t - \sin t) \; ; \; \varphi_2(t) = a(1 - \cos t)$

$\varphi'(t) = (a(1 - \cos t) \; ; \; a \sin t)$ $\quad \left[\sum x_i^2\right]$

$|\varphi'(t)| = a\sqrt{2}\,\sqrt{1 - \cos t}$

$l = a\sqrt{2} \int_0^{2\pi} \sqrt{1 - \cos t}$

$\int_0^{2\pi} \sqrt{1 - \cos t} \, dt = 2 \int_0^{\pi} \sqrt{1 - \cos 2z} \, dz =$

$= 2 \int_0^{\pi} \sqrt{1 - \cos^2 z + \sin^2 z} \, dz =$

$= 2\sqrt{2} \int_0^{\pi} \sin z \, dz = \underline{\underline{4\sqrt{2}}}$

$$M=\left\{(x,y,z)\in\mathbb{R}^3;\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}+\left(\frac{z}{c}\right)^{\frac{2}{3}}\leq 1\right\}$$

$$x=a\cdot\rho\cos^3\varphi\cos^3\omega$$

$$y=b\cdot\rho\sin^3\varphi\cos^3\omega$$

$$z=c\cdot\rho\sin^3\omega$$

$$G=\left\{(\rho,\varphi,\omega)\in\mathbb{R}^3;\rho>0;\varphi\in\left(0,\frac{\pi}{2}\right);\omega\in\left(0,\frac{\pi}{2}\right)\right\}$$

$$J=\begin{pmatrix} a\cdot\cos^3\varphi\cos^3\omega & -3a\rho\cos^2\varphi\sin\varphi\cos^3\omega & -3a\rho\cos^3\varphi\cos^2\omega\sin\omega \\ b\cdot\sin^3\varphi\cos^3\omega & 3b\rho\sin^2\varphi\sin\varphi\cos^3\omega & -3b\rho\sin^3\varphi\cos^2\omega\sin\omega \\ c\cdot\sin^3\omega & 0 & 3c\rho\sin^2\omega\cos\omega \end{pmatrix}$$

$$\det(J)=9abc\,\rho^2\sin^2\varphi\cos^2\varphi\sin^2\omega\cos^5\omega$$

$$Q=\{(x,y,z)\in\mathbb{R}^3;x>0,y>0,z>0\}$$

$$(\rho\cos^3\varphi\cos^3\omega)^{2/3}+(\rho\sin^3\varphi\cos^3\omega)^{2/3}+(\rho\sin^3\omega)^{2/3}\leq 1$$

$$L=\left\{(\rho,\varphi,\omega)\in\mathbb{R}^3;\rho\in(0,1\rangle;\varphi\in\left(0,\frac{\pi}{2}\right);\omega\in\left(0,\frac{\pi}{2}\right)\right\}$$

$$\iiint_{M\cap Q}dx\,dy\,dz=9abc\int_0^1\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\left(\rho^2\sin^2\varphi\cos^2\varphi\sin^2\omega\cos^5\omega\right)d\omega\,d\varphi\,d\rho=$$

$$=\frac{3\cdot 8}{35}abc\int_0^1\int_0^{\frac{\pi}{2}}\rho^2\sin^2\varphi\cos^2\,d\varphi\,d\rho=\frac{3\cdot 8\cdot\pi}{35\cdot 16}abc\int_0^1\rho^2\,d\rho=$$

$$=\frac{1}{35}\frac{\pi}{2}abc\quad\Longrightarrow\quad 8\cdot\frac{1}{35}\frac{\pi}{2}abc=\underline{\underline{\frac{4\pi abc}{35}}}$$

3) $\displaystyle\int_0^1\int_0^1 4x\, e^y \,dx\,dy =$

$$= \int_0^1\int_0^1 4x\, e^y \,dy\,dx =$$

$$= 4\int_0^1 x \int_0^1 e^y \,dy\,dx =$$

$$= 4\int_0^1 x \left[e^1 - e^0\right] dx =$$

$$\cancel{= 4\left[x e\right]_0^1 = 4(e-0) = 4e}$$

$$= 4\int_0^1 x\cdot(e-1)\,dx =$$

$A = (0,0) \quad B = (3,5)$

$\bar{\varphi}(t) = \left(t, \frac{5}{3} t\right)$

$t \in (0,2)$

$f(x)_s = x^2 \cos x$

$$l = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$\iint\limits_M (x^2+y^2)\,dx\,dy$$

$$M = \{(x,y) \in \mathbb{R}^2;\ |x|+|y| \leq 1\}$$

$\underline{x \in \langle -1;0\rangle:}$ $\quad M_x = \langle -x-1;\ x+1\rangle$

$\underline{x \in \langle 0;1\rangle:}$ $\quad M_x = \langle x-1;\ -x+1\rangle$

$$\iint\limits_M (x^2+y^2)\,dx\,dy = \int\limits_P \int\limits_{M_x} (x^2+y^2)\,dy\,dx =$$

$$= \int_{-1}^{0} \int\limits_{M_x} (x^2+y^2)\,dy\,dx + \int_{0}^{1} \int\limits_{M_x} (x^2+y^2)\,dy\,dx =$$

$$= \int_{-1}^{0} \int_{-x-1}^{x+1} (x^2+y^2)\,dy\,dx + \int_{0}^{1} \int_{x-1}^{-x+1} (x^2+y^2)\,dy\,dx =$$

$$= \int_{-1}^{0} \left[x^2 y + \frac{1}{3}y^3\right]_{-x-1}^{x+1} dx + \int_{0}^{1} \left[x^2 y + \frac{1}{3}y^3\right]_{x-1}^{-x+1} dx =$$

$$= \int_{-1}^{0} \left(2x^2(x+1) + \frac{2}{3}(x+1)^3\right) dx + \int_{0}^{1} \left(2x^2(1-x) + \frac{2}{3}(1-x)^3\right) dx =$$

$$= \frac{1}{3} + \frac{1}{3} = \underline{\underline{\frac{2}{3}}}$$

$$M = \left\{ (x,y) \in \mathbb{R}^2 ; y > \frac{x^2}{4} ; y < \frac{8}{4+x^2} \right\}$$

$$g_1 = \frac{x^2}{4} \quad ; \quad g_2 = \frac{8}{4+x^2}$$

$$g_1 = g_2 \Rightarrow A = (-2;1) ; B = (2,1)$$

$$\bar{M} = \left\{ (x,y) \in \mathbb{R}^2 ; x \in \langle -1;1 \rangle ; g_1(x) \leq y \leq g_2(x) \right\}$$

$$\int_{-2}^{2} \int_{g_1(x)}^{g_2(x)} dy \, dx = \int_{-2}^{2} \left(\frac{8}{4+x^2} - \frac{x^2}{4} \right) dx =$$

$$\int_{-2}^{2} \frac{8}{4+x^2} dx = 2\pi$$

$$\int_{-2}^{2} \frac{x^2}{4} dx = \frac{4}{3}$$

$$= 4\pi - \frac{4}{3} = \underline{2\left(\pi - \frac{2}{3}\right)}$$

$$\cancel{F(x,y)(x_1 = h_1, \dots)} = f(0) + \frac{1}{1!} f'(x,y) + \frac{1}{2!} f''(x,y)$$

$$T = f(0,0) + \frac{1}{1!}\left[\frac{\partial f(x,y)}{\partial x} h_1 + \frac{\partial f(x,y)}{\partial y} h_2\right] +$$

$$+ \frac{1}{2!}\left[\frac{\partial f(x,y)}{\partial x^2} h_1^2 + \frac{\partial f(x,y)}{\partial x \partial y} h_1 h_2 + \frac{\partial f(x,y)}{\partial y^2} h_2^2\right]$$

$$T(0,0) = f(0,0) + \frac{1}{1!}\left[\frac{\partial f(0,0)}{\partial x} h_1 + \frac{\partial f(0,0)}{\partial y} h_2\right] +$$

$$+ \frac{1}{2!}\left[\frac{\partial f(0,0)}{x^2} h_1^2 + \frac{\partial f(0,0)}{\partial x \partial y} h_1 h_2 + \frac{\partial f(0,0)}{\partial y^2} h_1^2\right]$$

$$f(x,y,z) = x + y + z$$

$$g(x,y,z) = \frac{1}{x} + \frac{1}{y} + \frac{1}{z} - 1 = 0$$

$$L(x,y,z) = x + y + z + \lambda\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} - 1\right)$$

$$L_x = 1 - \frac{\lambda}{x^2} = 0$$

$$L_y = 1 - \frac{\lambda}{y^2} = 0$$

$$L_z = 1 - \frac{\lambda}{z^2} = 0$$

$$L_\lambda = \frac{1}{x} + \frac{1}{y} + \frac{1}{z} - 1 = 0$$

$$x^2 - \lambda = 0 \Rightarrow x = \pm\sqrt{\lambda} = 3$$

$$y = \pm\sqrt{\lambda} = 3$$

$$z = \pm\sqrt{\lambda} = 3$$

$$\frac{3}{\pm\sqrt{\lambda}} = 1 \Rightarrow \pm\sqrt{\lambda} = 3$$

$$\lambda = 9$$

$$\det\begin{vmatrix} {\frac{2\lambda}{x^3}}^{F_{xx}} & 0^{F_{xy}} & 0^{F_{xz}} \\ 0^{F_{yx}} & {\frac{2\lambda}{y^3}}^{F_{yy}} & 0^{F_{yz}} \\ 0 & 0 & {\frac{2\lambda}{z^3}}^{F_{zz}} \end{vmatrix} = \frac{18}{3^{10}} > 0 \Rightarrow \text{P.D.} \Rightarrow \text{L.K.}$$

min.

$$\lim_{(x,y,z)\to\infty} e^{\overline{x^2+y^2+z^2}}$$

$$g(x,y,z) = x^2+y^2+z^2$$

$$s(x,y,z) = \frac{1}{g(x,y,z)}$$

$$\Rightarrow \lim_{s\to 0} e^{s} = \underline{\underline{1}}$$

TAYLER:

$$f(x+h) = f(x) + f'(x)\cdot h + \frac{f''(x)\,h^2}{2!} + \frac{f'''(x)\,h^3}{3!} + \dots + \frac{f^{(k)}(x)\,h^k}{(k)!}$$

$f(x,y) = e^x \cos(y)$

aprox. pol. 2. st. v $(\bar{0},\bar{0})$

$F(x+h_x, y+h_y) = F(x,y) + \nabla F(x,y)\bar{h} + \frac{\partial^2 f(x,y)\bar{h}}{2!}$

$\frac{\partial F}{\partial x} = e^x \cos y \to 1$

$\frac{\partial F}{\partial y} = e^x \sin y \to 0$

$\frac{\partial^2 F}{\partial x^2} = e^x \cos y \to 1$

$\frac{\partial^2 F}{\partial y^2} = -e^x \cdot \cos y \to -1$

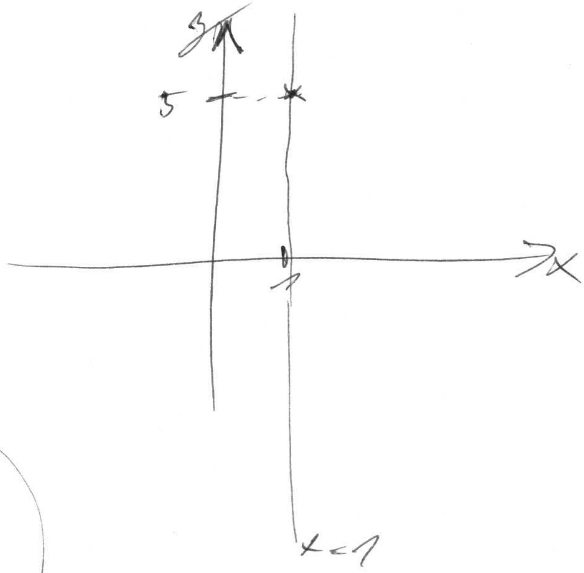
$\frac{\partial^2 F}{\partial x \partial y} = -e^x \sin y \to 0$

$d^2 f(x,y)(\bar{h}) = \bar{h}^T A \bar{h} = (h_x\ h_y) \cdot \begin{pmatrix} F_{xx} & F_{xy} \\ F_{yx} & F_{yy} \end{pmatrix} \begin{pmatrix} h_x \\ h_y \end{pmatrix} =$

$= h_x^2 F_{xx} + 2 h_x h_y \cdot F_{xy} + h_x^2 F_{yy}$

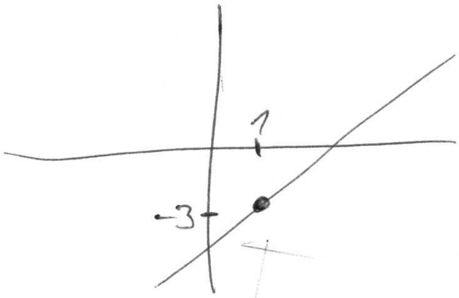
$F(h_x, h_y) = 1 + h_x + \frac{1}{2}(h_x^2 - h_y^2)$

$$\lim_{(x,y)\to(1,5)} \frac{1}{10x-2y}$$



$$\left.\begin{aligned} &\lim_{y\to 5_+} \frac{1}{10-2y} = +\infty \\ &\lim_{y\to 5_-} \frac{1}{10-2y} = -\infty \end{aligned}\right\} \quad \underline{\text{lim. neex.}}$$

$$\lim_{(x,y)\to(1,-3)} \frac{1}{3x+y}$$



$$y = kx - (k+3)$$

$$\frac{1}{3x+kx-(k+3)} = \frac{1}{x\cdot(3+k)-(k+3)}$$

$$\frac{1}{3x+y} = 0$$

$\underline{k=5}$: $\frac{1}{\ }$

$\underline{k=1}$: $\frac{1}{4x-4}$

lim nelt.

$\underline{k=0}$: $\frac{1}{3x-3}$

? −3 ? −9x

dokažte, že daná lim. neexistuje

~~tm~~ $f(x,y) = \frac{xy}{x^2+y^2}$

$D_f : \mathbb{R}^2 \setminus \{0,0\}$

$y = kx$ $\qquad$ $P_k = \{(x,y) \in \mathbb{R}^2 ; y = 2x\}$

$f(x, xk) = \frac{x^2 k}{x^2 + x^2 k^2} = \frac{k}{1+k^2}$

$\lim\limits_{\substack{x \to 0 \\ (x,y) \in P_k}} f(x, kx) = \frac{k}{1+k^2}$ $\Rightarrow$ nemá lim.

ex. lim ?

$$\lim_{(x,y)\to(0,0)} \frac{x^2 \cdot \sin\left(\frac{1}{x}\right)+y}{x+y} = ?$$

$$\lim_{y\to 0} \frac{x^2 \sin\frac{1}{x} + \overset{\nearrow 0}{y}}{\underset{\downarrow 0}{x+y}} = \frac{x^{\cancel{2}} \cdot \sin\frac{1}{x}}{\cancel{x}} = x \sin\frac{1}{x}$$

$$\lim_{x\to 0} x \cdot \sin\left(\frac{1}{x}\right) = 0$$

$$\lim_{x\to 0} \frac{x^2 \sin\frac{1}{x} + y}{x+y} = \frac{y}{y} = 1$$

$$\lim_{y\to 0} 1 = 1$$

$$\lim_{y\to 0}\left(\lim_{x\to 0} f(x,y)\right) = 0 \neq \lim_{x\to 0}\left(\lim_{y\to 0} f(x,y)\right) = 1$$

$\underline{m : 1}$

1) $\lim\limits_{(x,y)\to(0,0)} (x^2+y^2) \sin \frac{1}{xy}$

$\lim\limits_{y\to 0} (x^2+y^2) \sin \frac{1}{xy}$

$\hookrightarrow \sin \frac{1}{0} \to \infty$

$h(\bar{x}) \leq f(\bar{x}) \leq g(\bar{x})$

$-(x^2+y^2) \leq (x^2+y^2) \sin \frac{1}{xy} \leq (x^2+y^2)$

$\lim\limits_{(x,y)\to(0,0)} (x^2-y^2) = 0$

2) $\lim\limits_{(x,y,z)\to(3,2,1)} x\sqrt{2y^2+z^2} = ?$

$= 3 \cdot \sqrt{2\cdot 4+1} = 3 \cdot \sqrt{9} = \underline{9}$

[illegible]

① $\lim \left(1+x^2+y^2\right)^{\overline{(x^2+y^2)}}$

$$q = x^2+y^2$$

$$s = \frac{1}{x^2+y^2}$$

$$\lim_{s\to\infty}\left(1+\frac{1}{s}\right)^s = e$$

---

② $-2\lim_{(x,y)\to(0,1)} \underbrace{\arcsin\frac{1}{x-y}}_{-\frac{\pi}{2}}$

$\hookrightarrow \arcsin(-1)$

$$= -2\cdot\left(-\frac{\pi}{2}\right) = \underline{\underline{\pi}}$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin x^2 y^2}{(x^2+y^2)} = ?$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x} = ?$$

$$P_k = \{(x,y) \in \mathbb{R}^2 : y = kx \ \wedge k \neq 0\}$$

$$y = kx \quad \frac{\sin x^4 k^2}{x^2 + k^2 x^2} = \frac{\sin x^4 k^2}{x^4 (1+k^2)^2}$$

$$\lim_{x\to 0} \frac{\sin x^4 k^2}{x^4(1+k^2)^2} \underset{x^4=u}{=} \lim_{u\to 0} \frac{\sin u k^2}{u\cdot(1+k^2)^2} \overset{l'H}{=}$$

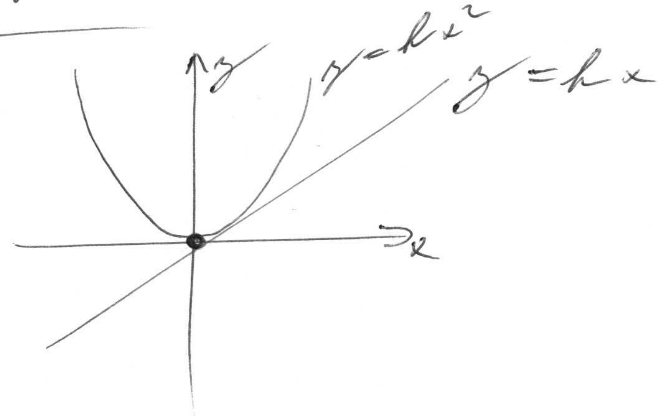
$$\overset{l'H}{=} \lim_{u\to 0} \frac{\cos(uk^2)k^2}{(1+k^2)^2} = \frac{k^2}{(1+k^2)^2}$$

$\Rightarrow$ <u>lim. neex.</u>

2. limita

$$f(x,y) = \frac{x^2 y}{x^4+y^2}$$

$D_f : \mathbb{R}^2 \setminus \{0,0\}$



$k \in \mathbb{R}$ P_k

$y = kx$:

$\lim f(x, kx) = ?$

$(x) \to 0$

$(x,y) \in P_k$

$$f(x, kx) = \frac{x^3 k}{x^4 + x^2 k^2} = \frac{xk}{x^2+k^2}$$

$$\lim_{\substack{x \to 0 \\ (x,y) \in P_k}} f(x, kx) = 0$$

$y = kx^2$:

$$f(x, kx^2) = \frac{x^4 k}{x^4 + k^2 x^4} = \frac{k}{1+k^2}$$

$$\lim_{\substack{x \to 0 \\ (x,y) \in Q_k}} f(x, kx^2) = \frac{k}{1+k^2}$$

$\Rightarrow f(x,y)$ nemá v $(0,0)$ lim.

najděte hodnotu fce defin. na

$D_f = R^2 \setminus \{0;0\}$

$$f(x,y) = \frac{x^3 - x^2 y + x y^2 - y^3}{x^2+y^2}$$

---

ověř dodef. fci v 0

$$F \begin{cases} \xrightarrow{(x,y)\neq(0,0)} f(x,y) \\ A \Leftarrow ? \end{cases} \Rightarrow \text{najít } A \text{, aby to bylo spojité}$$

$$\lim_{(x,y)\to(0;0)} f(x,y) = A$$

$x_n \to 0$ (používám $\bar{x}_n \to \bar{a} \Rightarrow f(x_n) \to \bar{b}$

$y_n \to 0$

$x_n - y_n \neq 0$

$$f(x_n,y_n) = \frac{x_n^3 - x_n^2 y_n + x_n y_n^2 - y_n^3}{x_n^2+y_n^2} =$$

$$= \frac{x_n(x_n^2+y_n^2) - y_n(x_n^2+y_n^2)}{x_n^2+y_n^2} =$$

$$= x_n - y_n \to 0$$

A volím $= 0$; hotovo

nakreslete vrstevnice zadané fce

$f(x,y) = \sqrt{y-x^2}$

$D_f: \; y - x^2 \geq 0 \;\Rightarrow\; y \geq x^2$

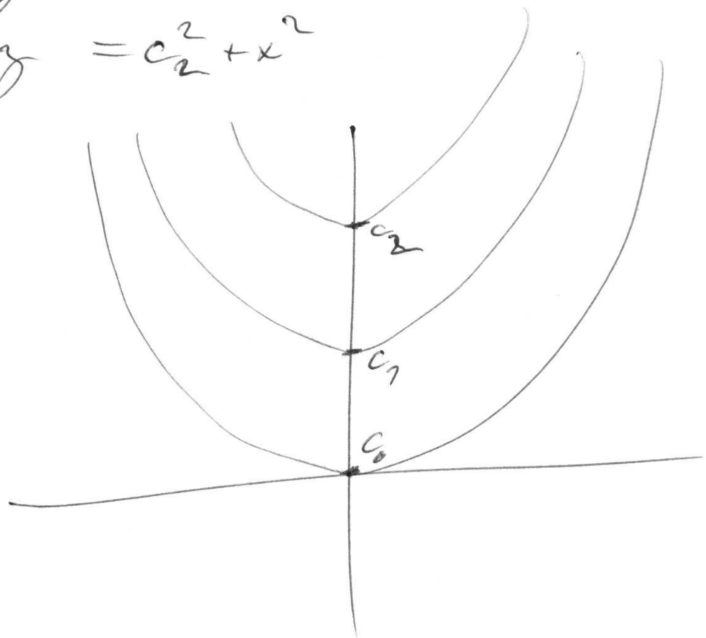
$f(x,y) = c_i \qquad i = 1 \ldots n$

$c_1 = \sqrt{y-x^2}$

$c_1^2 = y - x^2$

$y = c_1^2 + x^2$

$y = c_2^2 + x^2$



určete def. obor fce.

$$f(x,y,z) = \sqrt{z}\ \arcsin\left(1-x^2-y^2-z^2\right)$$

$$D_f(\sqrt{z}) = \{ z \in \mathbb{R} : z \geq 0 \}$$

$$D_f(\arcsin) = \{ u \in \langle -1; 1 \rangle \} \Rightarrow$$

$$\Rightarrow 1-x^2-y^2-z^2 \in \langle -1; 1 \rangle$$

$$\Downarrow$$

$$1-x^2-y^2-z^2 \geq -1$$
$$1-x^2-y^2-z^2 \leq 1$$

$$x^2+y^2+z^2 \leq 2$$
$$x^2+y^2+z^2 \geq 0$$

$$D_f = \{ (x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 \leq 2 \wedge z \geq 0 \}$$

$$f(x,y,z) = \frac{x^3 y}{z} \quad ; \quad z \neq 0$$

$$\frac{d f(x,y,z)}{dx} = \frac{3x^2 y}{z}$$

$$\left[\frac{x^3 y}{z} = x^3 y \cdot z^{-1} \right.$$

$$\frac{df}{dy} = \frac{x^3}{z}$$

$$\frac{df}{dz} = -\frac{x^3 y}{z^2}$$

$$f(x,y) = x^4 + y^4 - x^2 - 2xy - y^2$$

$$\frac{df}{dx}(x,y) = 4x^3 - 2x - 2y = 0$$

$$\frac{df}{dy}(x,y) = 4y^3 - 2x - 2x = 0$$

$(0,0)$
$(1,1)$
$(-1,-1)$

$$\frac{df}{dx\,dy} = -2$$

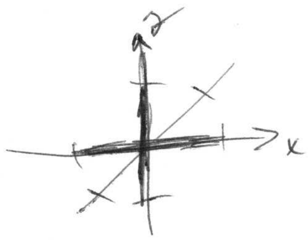
$$\frac{df}{dx\,dx} = 12x^2 - 2$$

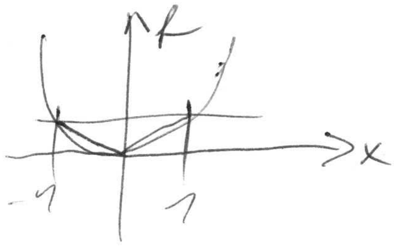
$$\frac{df}{dy\,dy} = 12y^2 - 2$$

$$A_{(1,1)} = \begin{pmatrix} 10 & -2 \\ -2 & 10 \end{pmatrix}$$ P.D

$$A_{(0,0)} = \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$$ není P.D.

$\Delta A = 0$





$f(x,y) = x^2 - y^2$? sedlový bod

? stac. bod $\Rightarrow$ gradient

$$\left.\begin{array}{l} \frac{df}{dx}(x,y) = 0 \Rightarrow 2x = 0 \\ \frac{df}{dy}(x,y) = 0 \Rightarrow -2y = 0 \end{array}\right\} \Rightarrow \begin{array}{l} x = 0 \\ y = 0 \end{array}$$

$$A = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \qquad \begin{array}{l} \Delta A_1 = 2 \\ \Delta A = -4 \end{array}$$

není P.D.

$-x^2 + y^2$

$$A = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix} \qquad \begin{array}{l} \Delta A_1 = -2 \\ \Delta A = -4 \end{array}$$

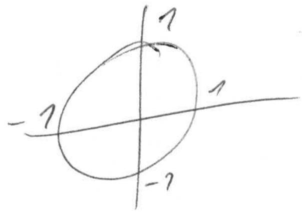
není N.D.

$\Rightarrow$ sedlový bod

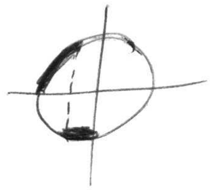
2 př.! $\;\; ? M$

$$f(x,y) = x^2 + y^2 + 1 = 0 \quad \Rightarrow M = \{\emptyset\}$$

$$f(x,y) = x^2 + y^2 \cancel{+} 1 = 0 \quad \Rightarrow \text{kružnice}$$

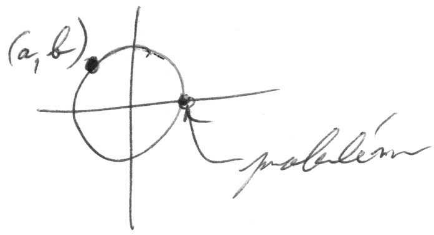


$$g(x) = \sqrt{1-x^2} \quad x \in \langle -1; C)$$
$$= -\sqrt{1-x^2} \quad x \in (C; -1\rangle$$



$\Rightarrow$ nekonečně mnoho $g(x)$

$\Rightarrow$ chci spojitost

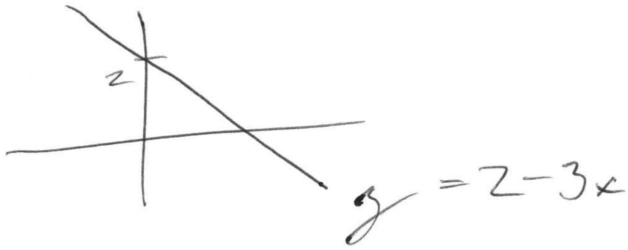


$$g(x) = \begin{cases} \sqrt{1-x^2} & b > 0 \quad f(a,b) = 0 \\ -\sqrt{1-x^2} & b < 0 \quad f(a,b) = 0 \end{cases}$$

Př ? množina bodů M

$f(x,y) = 6x + 2y - 4 = 0$

$y = 2 - 3x$



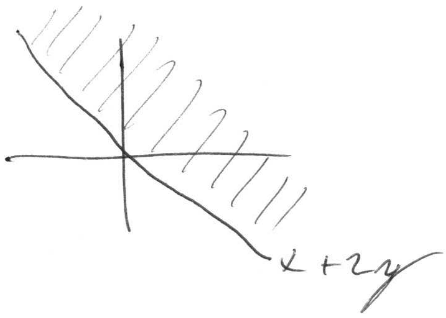
? $g(x)$, aby platila $f(x, g(x)) = 0$

$g(x) = 2 - 3x$

$f(x,y) = \sqrt{x^2 + y^2 + \cancel{4xy}} - x - 2y = 0$

$|x + 2y| = x + 2y$

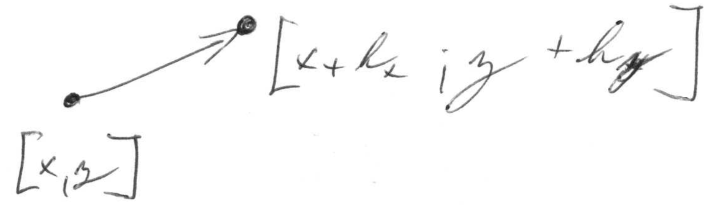
$M = ?$



$g(x)$ neex.

$$f(x,y) = x^2 + y^2$$

$$f(x+h_x ; y+h_y) = ?$$



$$f(x+h_x ; y+h_y) = f(x,y) + (2x \cdot h_x + 2y \cdot h_y) + (2h_x^2 + 2h_y^2)$$

$$\frac{df}{dx} = 2x \Rightarrow \frac{d^2f}{d_y d_x} = 0$$

$$\frac{df}{dy} = 2y \Rightarrow \frac{d^2f}{d_x d_y} = 0$$

$$\frac{d^2f}{d_x d_x} = 2 ; \frac{d^2f}{d_y d_y} = 2$$

? gradient v polárních souř.

$$x = r \cdot \cos\varphi$$
$$y = r \cdot \sin\varphi$$

$$\frac{df}{dx}(x, y) = ? \; ; \; \frac{df}{dy}(x, y) = ?$$

$$\frac{df}{dr}\Big(x(r, \varphi); y(r, \varphi)\Big) ; \frac{df}{d\varphi}\Big(x(r; \varphi); y(r; \varphi)\Big)$$

$$\frac{df}{dr} = \frac{df}{dx} \cdot \frac{dx}{dr} + \frac{df}{dy} \cdot \frac{dy}{dr}$$

$$\frac{df}{d\varphi} = \frac{df}{dx} \cdot \frac{dx}{d\varphi} + \frac{df}{dy} \cdot \frac{dy}{d\varphi}$$

$$\begin{pmatrix} \frac{df}{dr} \\ \frac{df}{d\varphi} \end{pmatrix} = \begin{pmatrix} \frac{dx}{dr} & \frac{dy}{dr} \\ \frac{dx}{d\varphi} & \frac{dy}{d\varphi} \end{pmatrix} \cdot \begin{pmatrix} \frac{df}{dx} \\ \frac{df}{dy} \end{pmatrix}$$

$$\frac{df}{dx} = \frac{\frac{df}{dr} \cdot \frac{dy}{d\varphi} - \frac{df}{d\varphi} \cdot \frac{dy}{dr}}{\frac{dx}{dr} \cdot \frac{dy}{d\varphi} - \frac{dx}{d\varphi} \cdot \frac{dy}{dr}}$$

$$\frac{df}{dy} = \frac{\frac{dx}{dr} \cdot \frac{df}{d\varphi} - \frac{df}{dr} \cdot \frac{dx}{d\varphi}}{\frac{dx}{dr} \cdot \frac{dy}{d\varphi} - \frac{dx}{d\varphi} \cdot \frac{dy}{dr}}$$