

Kvadratiky

Zadání

$$39x^2 + 6xy + 31y^2 + 318x + 86y + 659 = 0$$

Řešení

$$39x^2 + 3xy + 3xy + 31y^2 + 318x + 86y + 659 = 0$$

$$A = \begin{pmatrix} 39 & 3 \\ 3 & 31 \end{pmatrix} \quad \bar{b} = \begin{pmatrix} -159 \\ -43 \end{pmatrix} \quad c = 659$$

$$\begin{vmatrix} 39 - \lambda & 3 \\ 3 & 31 - \lambda \end{vmatrix} = 0$$

$$(39 - \lambda)(31 - \lambda) - 9 = 0$$

$$\lambda^2 - 70\lambda - 1200 = 0$$

$$\lambda_1 = \frac{70 - \sqrt{100}}{2}$$

$$\lambda_2 = \frac{70 + \sqrt{100}}{2}$$

$$\lambda_1 = 30$$

$$\lambda_2 = 40$$

$$\underline{\lambda_1 = 30}$$

$$\begin{pmatrix} 39 - 30 & 3 \\ 3 & 31 - 30 \end{pmatrix} \sim \begin{pmatrix} 9 & 3 \\ 3 & 1 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix} \sim \begin{pmatrix} 3 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\bar{x}_1 = (1; -3)^T / \sqrt{10} = \begin{pmatrix} \frac{1}{\sqrt{10}} \\ -\frac{3}{\sqrt{10}} \end{pmatrix}$$

$$\underline{\lambda_2 = 40}$$

$$\begin{pmatrix} 39 - 40 & 3 \\ 3 & 31 - 40 \end{pmatrix} \sim \begin{pmatrix} -1 & 3 \\ 3 & -9 \end{pmatrix} \sim \begin{pmatrix} -1 & 3 \\ 1 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 \\ 1 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 \\ 0 & 0 \end{pmatrix}$$

$$\bar{x}_2 = (3; 1)^T / \sqrt{10} = \begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{pmatrix}$$

Střed
 $A\bar{s} = \bar{b}$

$$\left(\begin{array}{cc|c} 39 & 3 & -159 \\ 3 & 31 & -43 \end{array} \right) \sim \left(\begin{array}{cc|c} 39 & 3 & -159 \\ 39 & 403 & -559 \end{array} \right) \sim \left(\begin{array}{cc|c} 13 & 1 & -53 \\ 0 & 400 & -400 \end{array} \right) \sim \left(\begin{array}{cc|c} 13 & 1 & -53 \\ 0 & 1 & -1 \end{array} \right)$$

$$\bar{s} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \Rightarrow \text{centrální}$$

$f(\bar{s})$

$$f(\bar{s}) = c - \bar{b} \cdot \bar{s} = 659 - (-159; -43) \cdot (-4; -1) = 659 - 679 = -20$$

$f(\bar{x})$

$$\lambda_1 x'^2 + \lambda_2 y'^2 + f(\bar{s}) = 0$$

$$30x'^2 + 40y'^2 - 20 = 0$$

$$30x'^2 + 40y'^2 = 20$$

$$\frac{3}{2}x'^2 + 2y'^2 = 1$$

$$\left(\frac{x'}{\sqrt{\frac{2}{3}}} \right)^2 + \left(\frac{y'}{\sqrt{2}} \right)^2 = 1$$

